The aim of this series is to provide a basic overview of Continuum Mechanics. If you have any doubts, you are advised to refer to the book 'Introduction to Continuum Mechanics', available from the EPFL bookshop and accessible online via campus wifi or VPN :
`https://ebookcentral.proquest.com/lib/epflch/detail.action?docID=452826`

1 Plate with a hole

In this exercise, we are interested in a quasi-infinite rectangular plate with a hole in it. Its behaviour is assumed to be linear isotropic elastic, and in plane stress conditions. The hole is centered on the origin of the reference frame and has a radius a. The plate is of thickness h. The plate is subjected to a static uniaxial stress $\sigma_{xx} = constant = p$ at its ends.

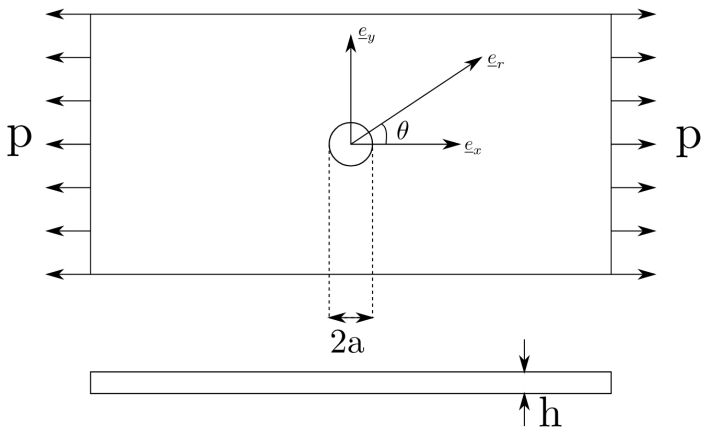

Figure 1: Hole plate subjected to uniaxial stress

The solution for an isotropic linear elastic material is known:

$$\sigma_{rr}(r,\theta) = \frac{p}{2}\left(1-\frac{a^2}{r^2}\right)\left[1+\left(1-3\frac{a^2}{r^2}\right)\cos 2\theta\right] \tag{1}$$

$$\sigma_{\theta\theta}(r,\theta) = \frac{p}{2}\left[1+\frac{a^2}{r^2}-\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta\right] \tag{2}$$

$$\tau_{r\theta}(r,\theta) = -\frac{p}{2}\left(1-\frac{a^2}{r^2}\right)\left(1+3\frac{a^2}{r^2}\right)\sin 2\theta \tag{3}$$

Question 1

Check that this solution satisfies the boundary conditions in terms of constraints. What's the equation that satisfies the 2D equilibrium ? (no need to develop it).

Question 2

Determine the maximum value of the radial stress $\sigma_{\theta\theta}$ and the position where it occurs.

Question 3

Determine the maximum value of the shear stress $\tau_{r\theta}$ and the position where it occurs.

Question 4

In the following exercises, we will only consider the state of stress at the edge of the hole ($r = a$).

Give the location and value of the maximum shear stress τ_{max}. On which plane does it act?

Question 5

What is the value of the stress intensity factor $K = \frac{\sigma_{max}}{\sigma_{xx}}$?

2 State of stress in a plate

A mild steel plate with stiffness $E = 30$ GPa and Poisson's ratio $\nu = 0.3$ is loaded with normal stresses σ_x and σ_y. An extensometer is bonded to the plate at an angle of $\phi = 30$ deg. The tension σ_x is equal to 18 MPa and the strain measured by the extensometer is $\epsilon = 407 \cdot 10^{-6}$.

If you have trouble answering the questions, you can refer to the hints at the end of the exercice, and to the following video: `https://www.youtube.com/watch?v=_DH3546mSCM`

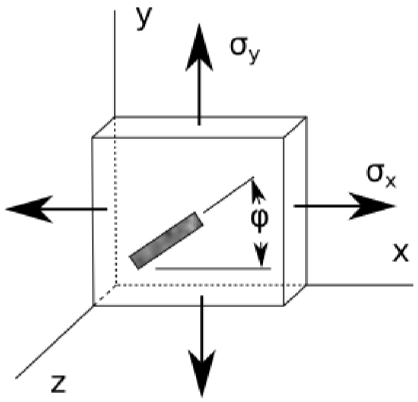

Figure 2: Plate with bonded extensometer

Question 1

Determine the value of the normal stress σ_y.

Hint : Try to relate the stresses to the strain measured by the extensometer.

Question 2

Determine the maximum tangential strain γ_{xy} and the maximum tangential stress τ_{xy}.

Question 3

What's the value of the maximum tangential strain γ_{yz} in the yz plane?

Question 4

Draw the Mohr's circle for the stress state.

Hints

In matrix form, Hooke's law for isotropic materials is written as:

$$\begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{23} \\ 2\epsilon_{13} \\ 2\epsilon_{12} \end{bmatrix} = \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix}$$

The following trigonometric identities can be useful:

$$\cos^2 \phi = \frac{1 + \cos 2\phi}{2}$$

$$\sin^2 \phi = \frac{1 - \cos 2\phi}{2}$$